DualHex

An abstract game for 2 players by Kashio Fujii

Components

- Four kinds of pieces: White Disc, White Cross, Black Disc, Black Cross
- A rhombus-shaped Hex board (any size, e.g. 9×9 or 11×11)
  Of the two pairs of opposite edges, one pair (marked white) belongs to White and the other (marked black) to Black.

Definitions

Color Whether a piece is White or Black.

Shape Whether a piece is a Disc (○) or a Cross (✕).

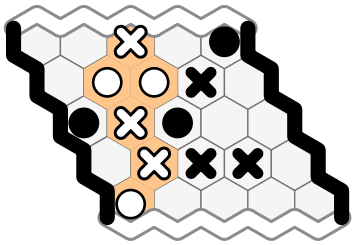
The White player owns the White color and the Disc shape; the Black player owns the Black color and the Cross shape.

Setup

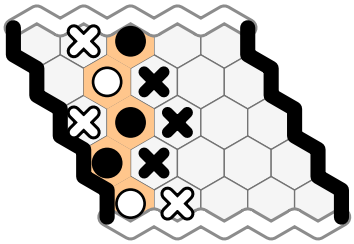
- The game starts with an empty board.
- Choose your color. White moves first.

End of the Game

- Check the winning conditions after every single piece placed.
  - **White wins** by connecting the two White edges with a chain of White pieces (of either shape), or with a chain of Discs (of either color).
  - **Black wins** by connecting the two Black edges with a chain of Black pieces (of either shape), or with a chain of Crosses (of either color).
- If a single piece completes both players' conditions at once, the player who placed it wins.
- One player always wins by the time the board fills up, so there are no draws.



White wins by color: White pieces (Discs and Crosses) join the two White edges (top and bottom).



White wins by shape: Discs of both colors join the two White edges. Black likewise wins by joining the two Black edges (left and right) with Black pieces or with Crosses.

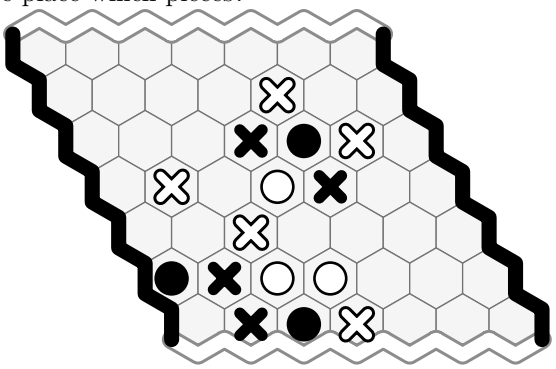
On Your Turn

On each turn, a player does the following.

1. **Place pieces (required)**
   Choose **two different kinds** out of the four, and place one of each on empty cells. **You may place pieces of your opponent's color**, not only your own. You may not place two pieces of the same kind in one turn. If the first piece decides the game, the game ends without placing the second.
2. **Exception on the first turn**
   On the very first turn of the game only, White places **just one piece: either a White Cross or a Black Disc**.

Example of Play

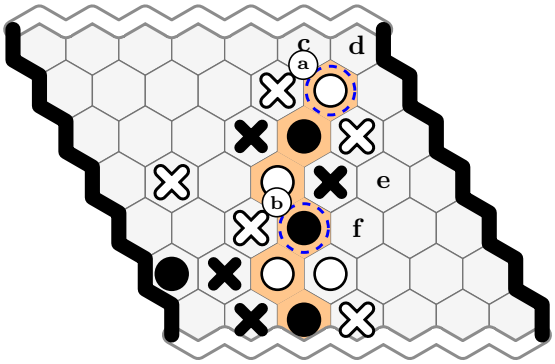
In the position below (on a 7×7 board) it is White's turn. In fact, White has already won. On which two cells should White place which pieces?



White's edges are the top and bottom, Black's the left and right.

Solution

The answer is a White Disc on a and a Black Disc on b.



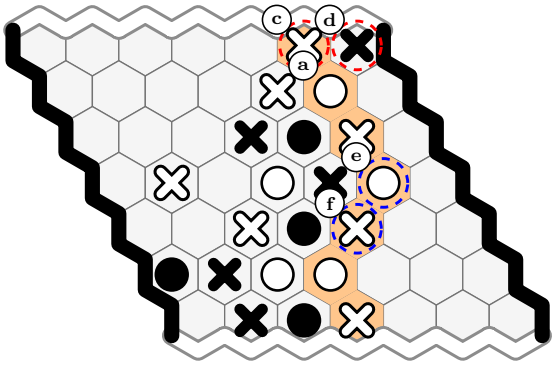
The dashed circles are the pieces placed. The shaded cells are the chain of Discs rising from the bottom edge.

With these two pieces, Discs (of either color) form one chain from the bottom edge up to a. The Disc on b cannot be a White Disc like the one on a, since you may not place two pieces of the same kind in one turn. So White uses a Disc of the opponent's color: a Black Disc.

From a, one more Disc on c or on d reaches the top edge. To stop this, Black must fill both c and d with Crosses (a Disc of either color there would complete White's chain).

But Black, too, cannot place two pieces of the same kind. To fill both cells, Black has to use one Black Cross and **one White Cross**.

If Black fills c and d



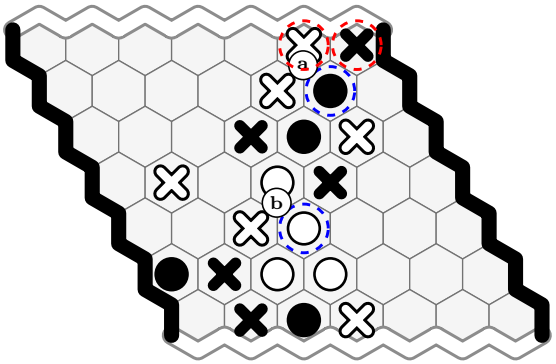
Red circles: Black's turn (a White Cross on c, a Black Cross on d). Blue circles: White's turn (a White Disc on e, a White Cross on f). The shaded cells are the chain of White pieces.

The White Cross that Black placed stands on White's top edge. White's **color** chain now runs from the top edge through c and a downward, and only e and f are missing. White places a White Disc on e and a White Cross on f, joins the top and bottom edges with White pieces, and wins.

If Black puts the two Crosses the other way round, the White Cross on d also touches a, and White wins in the same way.

Swapping the Discs' colors fails

A Black Disc on a and a White Disc on b make the same chain of Discs. But then, when Black fills c and d with a White Cross and a Black Cross, White cannot win: a is a Black piece, so the White Cross Black placed no longer helps White's color chain.



The failed attempt. Blue circles: White's turn. Red circles: Black's reply.

Which color you choose for a piece decides whether your opponent's reply ends up helping you: that was the way to win here.